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Numerical simulation of transit-time ultrasonic flowmeters: uncertainties due to flow profile and fluid turbulence

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Abstract

Flowmeter measurement using the ultrasonic transit-time method is based on the apparent difference of the sound velocity in the flow direction and in the opposite direction. This method gives a flow velocity averaged along a particular acoustical path. To convert this path velocity to a velocity averaged over the entire cross-section of the flowing medium, the knowledge of the flow velocity profile is essential. However, the acoustical paths joining the two transducers are supposed to be straight and fluid turbulence phenomena are neglected. In this paper, we describe a numerical procedure to estimate the uncertainties due to these approximations in the case of fully developed turbulence. The ultrasonic propagation is modelled in 2-D moving inhomogeneous media via a ray tracing algorithm. Influence of mean profiles of temperature and velocity is studied on simple examples. Fluid temperature fluctuations and fluid velocity turbulence are considered in the stochastic framework to obtain average uncertainties on the measurements of the liquid flow rate.

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1. Introduction

There is a great variety of ultrasonic techniques for measurement of liquid and gas flow. The transit-time ultrasonic flowmeters are based on the apparent difference of the sound velocity in the flow direction and in the flow opposite direction. Due to the simplicity of the measurement principle, this is one of the most common techniques in industrial applications [1]. This active method has the advantage to be non-intrusive with two transducers placed on the external surface of the pipe. From two ultrasonic transit times, the technique gives a flow velocity averaged along a particular acoustical path. To convert this path velocity to the velocity averaged over the entire cross-section of the flowing medium, the knowledge of the flow velocity profile is essential. Turbulence theory [2] predicts the pipe flow profile depending on the Reynolds number and on the pipe geometry. Therefore, the correction factor K_h, called meter factor [1] or hydraulic correction factor [3], is easily obtained.

A few sources of uncertainty (pipe geometry, pipe roughness, electronics, transit-time picking, flow profile) occur in this technique. Their estimation is an important subject of interest, in order to quantify their effects on flow estimation. Here we consider the problem of the flow profile. Its influence on ultrasonic flowmeter accuracy was recognized for a few decades [1]. However, even when the flow is steady and developed there remains some uncertainty about the profile, especially near the wall [4]. Estimating this uncertainty is essential to estimate the uncertainty on the hydraulic correction factor, and finally on the flow rate. Furthermore, some authors (e.g. [5,6]) try to reduce the influence of flow profile on the performance of flowmeters by using multipath quadrature solutions (i.e. weighted average over

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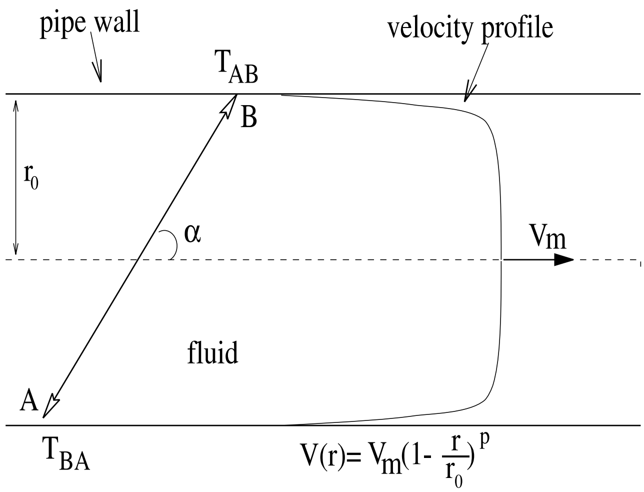

Fig. 1. Principle of the transit-time ultrasonic flowmeter method.

several chords at different distances from the pipe centerline).

In this paper, we consider flowmeters with the single midplane chord configuration (see Fig. 1). By supposing that the velocity profile is exactly known, we only study the influence of the application of the hydraulic correction factor to obtain the true flow rate. Indeed, a few phenomena related to the physics of the ultrasonic propagation are neglected in this process:

- Classically, the estimation of the hydraulic correction factor supposes straight rays, while the velocity profile deviates acoustical paths [7].
- Due to turbulence phenomena, the true velocity profile fluctuates around his mean profile in time and in space [2,8].
- The temperature is supposed to be homogeneous, while mean temperature gradients and turbulent thermal fluctuations also deviate the acoustical paths [7].

How to numerically analyse the effects on the fluid flow estimation of the mean temperature and velocity profiles and of the thermal and kinematic turbulences? As in [9], we propose to apply ray tracing techniques to simulate the acoustic propagation in moving inhomogeneous fluids. In our work, we restrict the problem to cylindrical pipes and the modelisation to 2-D media, for simplicity. The fluid turbulence is supposed to be fully developed, then the sensibility of the hydraulic correction factor with flow singularities is not studied. Moreover, the ultrasonic propagation in the pipe wall is not taken into account.

The following section presents the measuring principle of transit-time ultrasonic flowmeters. The modelling of high-frequency wave propagation in moving inhomogeneous media is considered in Section 3. In Section 4, numerical simulations of ultrasonic flowmeter are realized to quantify the effects of mean temperature and velocity profiles. A Monte Carlo method, based on the statistical representation of the thermal and kinematic turbulence phenomena gives, in Section 5, some estimations of the flow measurement bias in relation to the turbulence parameters.

2. Transit-time ultrasonic flowmeter

The geometry of the transit-time method is represented in Fig. 1. Two ultrasonic transducers (supposed to be point like and located on A and B) send and detect a short sound pulse with an oblique propagation direction (angle α with the pipe axis). The upstream signal (from B to A) is delayed, and the downstream signal (from A to B) is speeded up by the moving fluid. The transit time t_{AB} of the upstream signal and the transit time t_{BA} of the downstream signal are measured.

If the flow is axially uniform (velocity v) and the acoustical celerity c is constant in the fluid (constant temperature field), the flow velocity can be estimated by

$$v = \frac{r_0}{\sin\alpha\cos\alpha}\left(\frac{1}{t_{AB}} - \frac{1}{t_{BA}}\right) = \frac{r_0 \Delta t}{\sin\alpha\cos\alpha\, t_{AB} t_{BA}}, \quad (1)$$

where r_0 is the pipe radius and $\Delta t = t_{BA} - t_{AB}$ is called the transit-time differential [1].

However from turbulence theory, we know that the velocity of fluid flow is not homogeneous. Prandtl's law [2] describes the mean velocity profiles depending on the Reynolds number

$$Re = \frac{v_{moy} D}{\nu}, \quad (2)$$

where v_{moy} is the average velocity over the tube cross-section, D is the tube diameter, and ν is the kinematic viscosity ($\nu = 8 \times 10^{-7}$ m^2/s for pure water at 30 °C). In the domain of turbulent flow ($Re > 4000$), the velocity profile is called the "pipe profile" and is given by [2]

$$v(r) = v_m \left(1 - \frac{r}{r_0}\right)^p, \quad (3)$$

where r is the distance from the axis, v_m is the velocity on this axis (Fig. 1), and p is a parameter depending on the Reynolds number. In this paper, we restrict our problem to turbulent flows with the velocity profile (3). In our industrial context, p is chosen as in [3]:

$$p = 0.25 - 0.023 \log_{10} Re. \quad (4)$$

In the case of pure water at temperature 30 °C, in pipes with $D = 0.1$ m, the turbulent domain supposes that $v_{moy} > 0.03$ m/s.

The velocity v given by Eq. (1) corresponds to the fluid velocity averaged on the acoustical path AB, supposed to be a straight line. The integration of the velocity field (3) on the straight line AB gives

$$v = \frac{v_m}{1+p}. \quad (5)$$

To calculate the flow rate Q, one needs to consider the velocity averaged over the tube cross-section v_{moy}:

$$Q = \pi r_0^2 v_{moy}. \quad (6)$$

By integration of the velocity field (3) over the cylindrical tube cross-section, we find

$$v_{moy} = \frac{v_m}{(1+p)\left(1+\frac{p}{2}\right)}. \quad (7)$$

Therefore, to obtain v_{moy} from v, we just apply the hydraulic correction factor $K_{\rm h}$, which writes

$$K_{\rm h} = \frac{v}{v_{moy}} = 1 + \frac{p}{2}. \quad (8)$$

Remark that this factor increases with decreasing Reynolds number.

3. Modelling high-frequency wave propagation

For industrial applications, ultrasonic frequencies of transit-time flowmeters are of the order of 1 MHz. In pure water at 30 °C, the acoustical velocity is equal to $c_0 = 1509.126858$ m/s. Therefore, the acoustic wavelengths are of the order of $\lambda \sim 1.5$ mm. From wave theory, we know that travel times are defined in the theory of geometric acoustics [7]. This theory is valid under high-frequency hypotheses [10,11]:

$$\lambda \ll L \quad \text{and} \quad \sqrt{\lambda X} \ll L, \quad (9)$$

where L is the typical spatial size of the acoustic velocity fluctuations and X is the wave propagation distance. In practice, these conditions express [12,13]:

$$\lambda < \frac{L}{2} \quad \text{and} \quad \sqrt{\frac{\lambda X}{2}} < L. \quad (10)$$

The numerical simulation of high-frequency acoustic waves is usually done via a ray tracing algorithm [7]. In a recent work [9], the ray tracing method has been implemented to simulate ultrasonic flowmeters with transducers of finite surface. This technique would require a lot of computation time in the case of point like source and receiver (which is the assumption in our study), because it would be necessary to find the ray that joins "exactly" the two points (eigen ray).

The Gaussian beam approach [14–16] solves the wave equation in the neighborhood of the conventional rays using the parabolic approximation. The solution associates with each ray a beam having a Gaussian amplitude normal to the ray. The solution for a given source is then constructed by a superposition of Gaussian beams along nearby rays. The rays are given from the standard ray tracing technique, but the use of Gaussian beams has the advantage to avoid eigenray computations to find the travel times at fixed receivers. We use the same formulation than in [16] where this technique is used to simulate the propagation of acoustic waves in moving media (wind in the atmosphere).

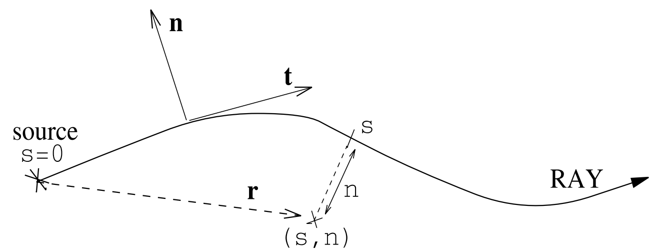

Fig. 2. Ray centred orthogonal basis $(\mathbf{t}, \mathbf{n})$ and coordinate system (s, n); $\mathbf{t}$ and $\mathbf{n}$ are respectively the unit vectors tangent and perpendicular to the ray, s is the arclength along the ray.

First, we solve the ray tracing system in the moving medium. Let us denote $\mathbf{r}(x,z)$ the ray position vector, $\mathbf{t}(\mathbf{r})$ the unit vector tangent to the ray (Fig. 2), $\tau(\mathbf{r})$ the travel time of the wave on the ray, $c(\mathbf{r})$ the acoustic velocity, $\mathbf{v}(\mathbf{r})$ the fluid velocity vector, $\mathbf{S} = (\mathbf{t}/(c + \mathbf{t}\cdot\mathbf{v}))$ the slowness vector and $\Omega = 1 - \mathbf{v}\cdot\mathbf{S}$. In moving media, the ray tracing system is usually written in function of the time τ [7]. We prefer to express it in function of the arc length s along the ray (change of variable $d\tau = ds\,(\Omega/c)$):

$$\begin{cases} \dfrac{d\mathbf{r}}{ds} = c\mathbf{S} + \dfrac{\Omega}{c}\mathbf{v}, \\ \dfrac{d\mathbf{S}}{ds} = -\dfrac{\Omega^2}{c^2}\nabla c - \dfrac{\Omega}{c}\sum_{i=x,z} S_i \nabla v_i, \\ \dfrac{d\tau}{ds} = \dfrac{\Omega}{c}. \end{cases} \quad (11)$$

Secondly, we solve, along each ray, the dynamic ray tracing system which writes [14,16]:

$$\begin{cases} \dfrac{dq}{ds} = cp, \\ \dfrac{dp}{ds} = -\dfrac{d^2 c}{dn^2}\dfrac{q}{c^2}, \end{cases} \quad (12)$$

where p and q are two complex-valued function and $\mathbf{n}(\mathbf{r})$ is the unit vector perpendicular to the ray. Each ray defines a ray centred coordinate system (Fig. 2), where a point is localized by (s, n) with n the distance of the point to the ray (perpendicular projection) and s the curvilinear abscissa of this projection. The travel time $t(s, n)$ is then given by finding the nearest ray to the receiver (corresponding to the smallest projection distance) and by applying the formula

$$t(s,n) = \tau(s) + \Re\left[\frac{p(s)}{q(s)}\right]\frac{n^2}{2}, \quad (13)$$

where τ is the travel time on the ray and $\Re(\cdot)$ represents the real part function.

In practice, we solve the systems (11) and (12) by a second order Runge–Kutta scheme. A fan of rays is launched in an opening angle θ to obtain a sufficient density of rays. The key parameter is the chosen step of the curvilinear abscissa ds. In the application of the next section, we adjust the parameters to obtain a sufficient accuracy on the transit times: we trace 50 rays in a cone $\theta = 5°$ with a step $ds = D/1000$ (D is the pipe diameter).

Remark. The velocity profile (3) has infinite derivatives for $r = r_0$. This causes a problem to trace rays from the surface of the pipe (where $r = r_0$) because the derivatives of **v** are used in the ray system (11). We avoid this difficulty by placing the ultrasonic transducers at very small distances of the edges. We therefore take into account this modification in Eqs. (5) and (8).

4. Uncertainties due to mean profiles

In this section, we realize some numerical simulations to quantify the effects of the velocity and temperature profiles on the ultrasonic flow measurement. For each simulation, the fixed parameters are the pipe diameter D, the ultrasonic propagation angle α, the flow rate Q and the temperature field T. The temperature field is linked to the acoustic velocity field c by the classical relation valid for pure water [17]. We choose for α a typical value in some industrial applications (steel pipe containing water with clamp-on transducers): $\alpha = 20°$.

Our numerical procedure consists in a few steps:

- derivation of the parameters v_{moy} by Eq. (6) and Re by Eq. (2);
- derivation of the parameters p and v_m of the velocity profile (pipe profile) by Eqs. (4) and (7);
- calculation of the transit times t_{AB} and t_{BA} by simulation of the ultrasonic propagation using the ray tracing and Gaussian beam methods;
- application of formula (1) to obtain v;
- estimation $\hat{v}_{moy}$ of v_{moy} and $\hat{Q}$ of Q with formulas $\hat{v}_{moy} = v/(1 + (p/2))$ (Eq. (8)) and $\hat{Q} = \pi r_0^2 \hat{v}_{moy}$;
- calculation of the relative error (in %) on the flow rate estimation:

$$\frac{dQ}{Q} = \frac{\hat{Q} - Q}{Q} \times 100. \tag{14}$$

In Fig. 3, we present results concerning the influence of the velocity profile (taking four different values) on the estimation error on Q for different pipe diameters D. We plot curves in function of the averaged velocity v_{moy} varying between 0.1 and 50 m/s (with a logarithmic scale). In industrial applications, the maximum fluid velocity arises to 15 m/s. However it seems to be interesting to look at the effects for larger values. In all the range of v_{moy} values, the relative errors corresponding to $D = 0.2$, 0.5 and 1 m vary between 0.3% and 0.4%. The three curves have a similar behaviour in function of v_{moy}: an increase in the range 0.1 m/s $< v_{moy} <$ 3 m/s, then a low decrease. The increase is coherent with the fact that the ray bending increases when the fluid velocity increases. However the pipe profile has also an influence on the acoustical paths: when the Reynolds number increases, the parameter p (restricted in the range $0 < p < 0.2$) decreases and the pipe profile tends to a uniform profile. This is coherent with the decrease of the estimation error.

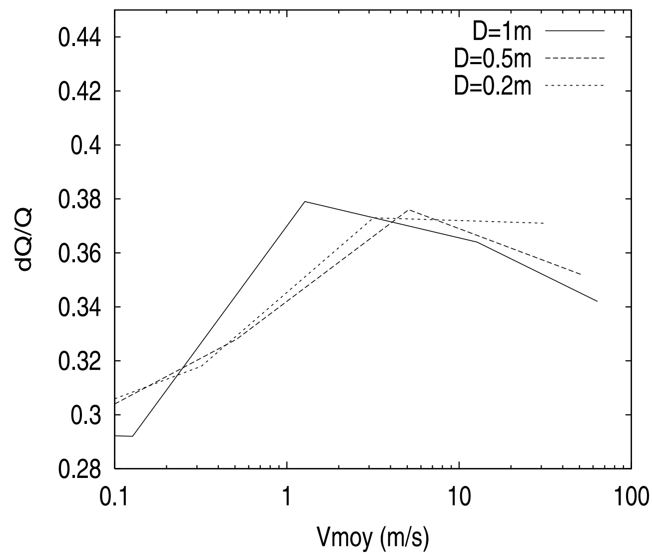

Fig. 3. Relative errors of $\hat{Q}$ due to mean velocity profiles (pipe profiles) for three different pipe diameters. The temperature field is constant ($T = 30$ °C).

In Fig. 4, we present the results of three simulations concerning the influence of the temperature field on the estimation error on the flow rate. We perform tests for a velocity profile (3) with a diameter $D = 0.5$ m. For the constant temperature fields ($T = 30$ and 80 °C), we

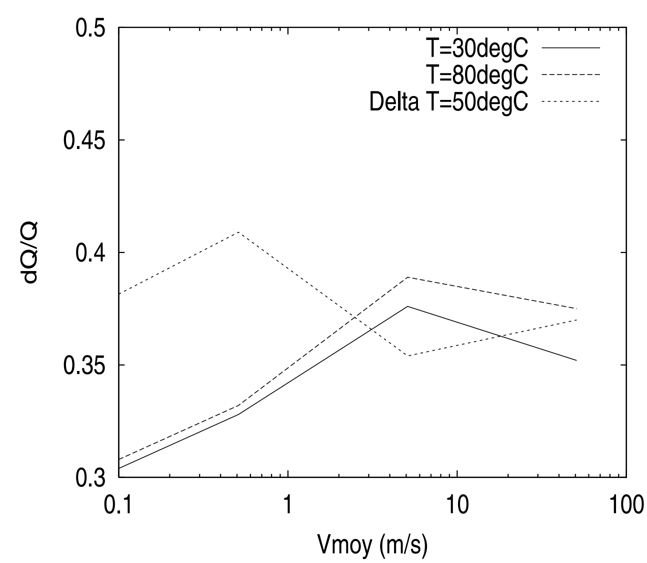

Fig. 4. Relative errors of $\hat{Q}$ due to pipe velocity profile ($D = 0.5$ m) and for different mean temperature profiles: two constant fields ($T_0 = 30$ °C, $T_0 = 80$ °C), one with a constant vertical gradient (T(up) $= 80$ °C, T(down) $= 30$ °C).

observe a regular increase of (dQ/Q) from 0.3% to 0.4% in the range $0.1\ \text{m/s} < v_{moy} < 5$ m/s. For larger values of v_{moy}, (dQ/Q) stabilizes between 0.35% and 0.4%. For the temperature field with strong gradient (50 °C between up and down), we observe that (dQ/Q) varies also between 0.35% and 0.4%.

We conclude from these tests that the flow rate is overestimated ($(dQ/Q) > 0$) and the bias depends essentially on the averaged fluid velocity v_{moy} of the pipe velocity profile. In our specific configuration, flowmeter errors are of the order of 0.35%. Finally, mean temperature profile has a small influence compared to the effect of the pipe profile.

5. The thermal and kinematic turbulence effects

We have studied in the previous section the effects of mean temperature and velocity profiles. However spatially small ($<D$) fluctuations are not introduced into this deterministic representation. The stochastic modelisation of small temperature and velocity fluctuations can take into account the fully developed turbulence phenomena [8]. Therefore, large trends are incorporated into the deterministic component and other small perturbations are described in stochastic fields.

The main hypotheses of this modelisation are the temporal stationarity and the statistical spatial homogeneity and isotropy of these fluctuations. Usually, medium fluctuations have a zero mean and are represented by a few statistical parameters: a correlation function $N(\mathbf{r})$ (statistical structure), a standard-deviation σ (fluctuations strength) and a correlation length L (typical size of heterogeneities). In this study, we choose a Gaussian correlation function for the stochastic fluctuations: $N(\mathbf{r}) = \sigma^2 e^{-(r^2/L^2)}$. This is the more convenient framework for first feasibility tests. In Fig. 5, we visualize realizations of thermal (up) and kinematic (down) turbulent fields with Gaussian correlation functions. The deterministic component ($T_0 = 30$ °C) of the temperature field is perturbed by small random fluctuations, while the velocity field fluctuates randomly around its horizontally uniform component ($v_{0x} = 5$ m/s, $v_{0z} = 0$).

The propagation of acoustic, electromagnetic or elastic waves in turbulent media is a subject of great interest for several decades [18]. For the numerical simulation of such phenomena, the followed method uses the random Fourier modes for generating random fields [19,20]. Usually, the high-frequency acoustic wave propagation in turbulent media is simulated via the ray tracing technique [19] or the parabolic wave equation [20]. The frozen medium hypothesis (approximation of low Mach number) permits to neglect the fluid evolution (kinematic and thermal heterogeneities) during the ultrasonic propagation. In [21–23], the technique of Gaussian beams has been applied to modelize wave propagation in motionless random media (thermal heterogeneity only). For this work, we have verified that the Gaussian beam summation method can be applied to simulate the wave propagation in moving inhomogeneous random media (kinematic and thermal turbulence).

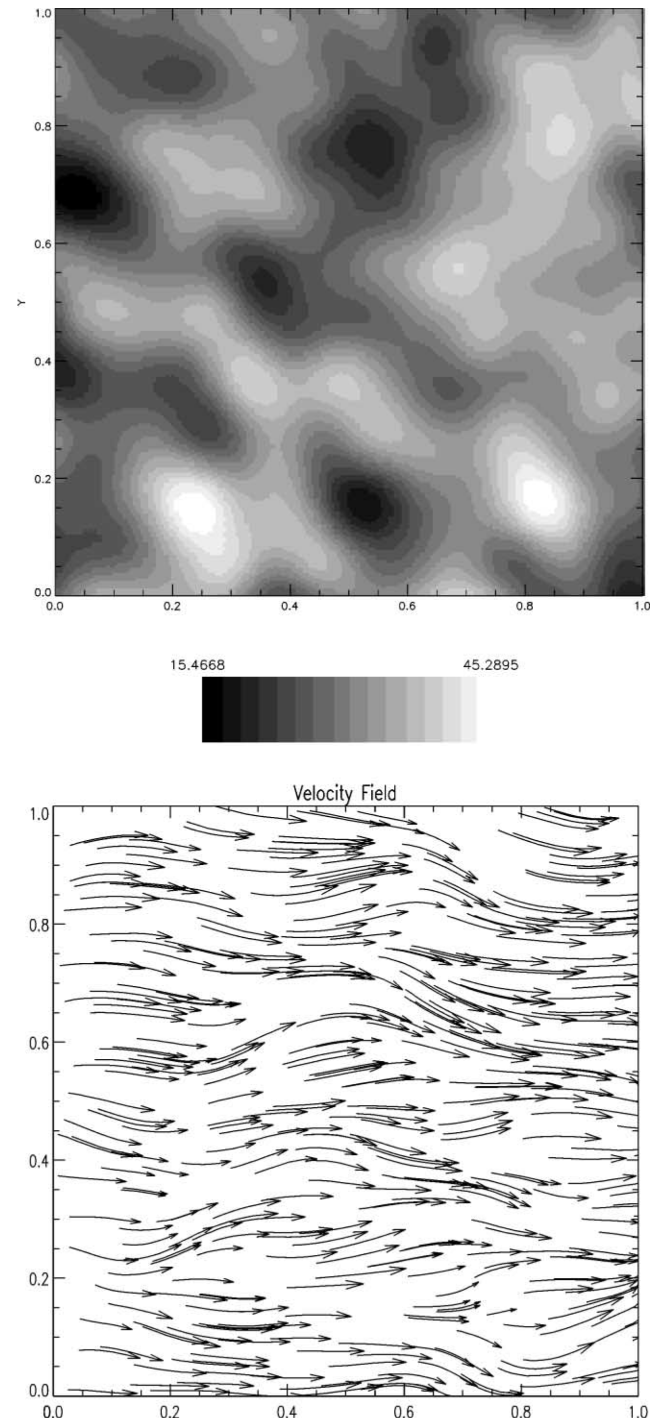

Fig. 5. Realizations of turbulent fields (axis in metres). Up: temperature fluctuations ($T_0 = 30$ °C, Gaussian correlation, $L = 0.1$ m, $\sigma_T^2 = \sqrt{\langle T^2 \rangle} = 5$ °C. Down: fluid velocity fluctuations ($\mathbf{v}_0 = (5, 0)$ m/s, Gaussian correlation, $L = 0.1$ m, $\sigma_v^2 = \sqrt{\langle v_x^2 \rangle} = \sqrt{\langle v_z^2 \rangle} = 1$ m/s).

Table 1
Relative errors (in %) on the estimation of flow rate ($D = 1$ m) with thermal and kinematic turbulences (Gaussian correlation, $L = 0.1$ m): $((\hat{Q} - Q)/Q) \times 100$

v_{moy} (m/s)	Turbulence velocity profile	No turbulence	Thermal $\sigma_T = 5$ °C	Kinematic $(\sigma_v/v_{moy}) = 0.08$	Thermal + kinematic $\sigma_T = 5$ °C, $(\sigma_v/v_{moy}) = 0.08$
13	Uniform	0.009	0.22	0.60	0.83
13	Pipe flow	0.36	0.53	0.95	1.16
1.3	Uniform	0.009	1.26	0.58	1.86
1.3	Pipe flow	0.38	1.54	0.95	2.14

The Monte Carlo technique, that we use, consists to repeat the numerical experience of flowmeter measurements in a large number of turbulent realizations, and to calculate the mean effect on the flow estimation:

$$\hat{Q} = \pi r_0^2 \frac{v}{K_h} = \frac{\pi r_0^3}{K_h \cos\alpha \sin\alpha} \left\langle \frac{1}{t_{AB}} - \frac{1}{t_{BA}} \right\rangle, \quad (15)$$

where the brackets $\langle \cdot \rangle$ denote the averaging on the measurement $(1/t_{AB}) - (1/t_{BA})$ of all the realizations. At each step, we randomly simulate one realization of the turbulent field (temperature or velocity), we add these fluctuations to the deterministic profiles, and we calculate the ultrasonic transit times t_{AB} and t_{BA} inside the medium.

In the following tests, the pipe diameter is $D = 1$ m. For the mean velocity profiles, we have chosen four cases: high uniform profile ($v_{moy} = 13$ m/s), high pipe profile ($v_{moy} = 13$ m/s), weak uniform profile ($v_{moy} = 1.3$ m/s) and weak pipe profile ($v_{moy} = 1.3$ m/s). In Table 1, the column with no turbulence reminds the influence of the mean velocity profiles. If the velocity profile is uniform, there is only a bias due to the numerical discretization (0.009%). With the pipe velocity profile (3), the estimation errors are of order 0.35% (calculated in the previous section).

For the turbulent cases, we study the effects of thermal differentials of ten degrees order ($\sigma_T = 5$ °C) and the effects of kinematic heterogeneities smaller than the mean velocity (the standard deviation is approximately one tenth of v_{moy}). The correlation length of the heterogeneities is $L = 0.1$ m; then spatial typical size of the heterogeneities is one tenth of the diameter. We take 200 realizations of the turbulence field because we have observed that the convergence of $\hat{Q}$ is obtained between 100 and 200 realizations. The interpretation of Table 1 is presented below:

- The error due to the thermal heterogeneities is much larger for the weak velocity field (1.26%) than for the high velocity field (0.22%). The fluid velocity tends to decrease the effects of the temperature.
- The errors due to the kinematic heterogeneities are the same for the weak velocity field and for the high velocity field (0.6% for uniform flow). Indeed, the ratio between the turbulence standard deviation and the average velocity is the same ($\sigma_v/v_{moy} = 0.08$).
- The error in the pipe flow case with a thermal or kinematic turbulence is approximately the sum of the error due to the mean pipe profile (with no turbulence) and the error due to the thermal or kinematic turbulence (with the uniform profile).
- The error in the pipe flow case with the combined turbulences (thermal and kinematic) is approximately the sum of the error due to the mean pipe profile (with no turbulence), the error due to the thermal turbulence (with the uniform profile), and the error due to the kinematic turbulence (with the uniform profile).

Finally, the flow estimation bias is always positive (overestimation of the flow rate). In the most difficult situation (weak flow, pipe velocity profile, thermal and kinematic turbulences), it can reach 2%.

6. Conclusion

Hydraulic turbulence phenomena appear in a large variety of industrial pipe flows. They introduce some errors in the ultrasonic flowmeter measurements. In this study, we have described a numerical procedure to quantify the transit-time flowmeter uncertainties caused by the deviation of the acoustical paths from the straight lines. In our simplified configuration, we have found that the flow rate is overestimed of 0.35% due to the effects of the mean velocity profile, and that the mean thermal field has a negligible influence.

Small fluid heterogeneities are modelized in the stochastic framework. This permits to treat the problem via a Monte Carlo method which gives an average estimation error from the repetition of the measurements in different realizations (chosen randomly). We obtain in our tests a positive bias of 1% or 2% order.

Of course, our 2-D propagation model limits the interpretation of the results. For example, we know that in a 3-D modelisation, the effects of the mean and turbulent fields are theoretically stronger on the travel times [12,23]. Moreover our idealized stochastic model is far from real applications. The next step of this work will be to numerically simulate the ultrasonic flowmeters in more realistic synthetic fields. For example, we could use non-Gaussian correlation function for the medium fluctuations (e.g. Kolmogorov model, Von Karman, ... [23]). Furthermore, a deterministic fluid mechanics code can furnish, for a specific industrial application, realistic temperature and velocity fields in different parts of the pipe.

Acknowledgement

In this study, we have used and extended the code developed by David Fiorina during his thesis work [21] at the Commissariat à l'Énergie Atomique.

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